

ON GLOBALLY PARA FRAMED METRIC MANIFOLD

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Abstract— In this paper, I have defined General F-connexion, O*-F-connexion and various properties have discussed therein. Theorems related to these connexions have also been stated and proved.

Keywords : F-connexion, O*-F-connexion Para framed, Metric Manifold, Connexion,

I. INTRODUCTION

Let M_n ($n = r+s$, r even), be a manifold with F-structure of rank r . Let there exist on M_n , s vector fields U and s 1-forms u , such that

$$\overline{X} - X = -u(X)U, \tag{1.1a}$$

Where

$$\overline{X} \stackrel{def}{=} FX, \tag{1.1b}$$

$$\overline{U} = 0 \tag{1.1c}$$

$$u(\overline{X}) = 0, \tag{1.1d}$$

$$u(U) = \delta = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}. \tag{1.1e}$$

Then $\{F, U, u\}$ is called Globally Para Framed F-structure and M_n is said to be a Globally Para Framed F-manifold or simply a globally Para Framed manifold.

Let there exist on M_n a Riemannian metric g , such that

$$g(\overline{X}, \overline{Y}) = g(X, Y) - u(X)u(Y), \tag{1.2a}$$

$$u(X) \stackrel{def}{=} g(X, U). \tag{1.2b}$$

Then $\{F, U, u, g\}$ is said to be Para Framed metric structure and the manifold M_n is called Para framed metric manifold.

A bilinear function A in M_n is said to be pure in the two slots X and Y , if

$$A(\overline{X}, \overline{Y}) + A(X, Y) = 0. \tag{1.3a}$$

A bilinear function A in M_n is said to be hybrid in the two slots X and Y , if

$$A(\overline{X}, \overline{Y}) - A(X, Y) = 0. \tag{1.3b}$$

Let us put

$$'F(X, Y) \stackrel{def}{=} g(\overline{X}, Y). \tag{1.4}$$

Then the following equations hold:

$$'F(X, Y) = -'F(Y, X). \tag{1.5a}$$

This shows that $'F$ is skew-symmetric in X and Y .

$$'F(\overline{X}, Y) = 'F(X, \overline{Y}), \tag{1.5b}$$

$$'F(\overline{X}, \overline{Y}) = -'F(X, Y). \tag{1.5c}$$

This shows that $'F$ is pure in X and Y .

II. GENERAL F- CONNEXION

A connexion D in M_n is called a general F-connexion, if

$$(D_X F)Y = 0, \tag{2.1a}$$

which is equivalent to

$$(D_X \overline{Y}) = \overline{D_X Y}. \tag{2.1b}$$

Theorem (2.1). For general F-connexion in M_n , we have

$$u(Y)(D_X U) + (D_X u)(Y)U = 0, \tag{2.2a}$$

$$D_X (u(Y)U) = u(D_X Y)U, \tag{2.2b}$$

$$D_X u(Y)U = 0 = D_{\overline{X}} u(Y)U, \tag{2.2c}$$

$$(D_X u)(Y) + u(Y)u(D_X U) = 0, \tag{2.2d}$$

$$(D_X u)(U) + u(D_X U) = 0, \tag{2.2e}$$

$$(D_U^x u)(U) + u(D_U^x U) = 0, \quad (2.2)f$$

$$(D_U^x F)Y = 0, \quad (2.2)g$$

$$D_U^x \bar{Y} = \overline{D_U^x Y}, \quad (2.2)h$$

$$u(D_X^x \bar{Y})U = 0 = u(D_X^x \bar{Y})U. \quad (2.2)i$$

Proof. Barring Y in equation (2.1)b then using the equations (1.1)a and (2.1)b in the resulting equation, we get the equation (2.2)a. Barring the equation (2.1)b and using the equations (2.1)b and (1.1)a, we get the equation (2.2)b. Barring the equation (2.2)b throughout and using (1.1)c, we obtain the equation (2.2)c. Applying u on (2.2)a and using the equation (1.1)e, we get (2.2)d. Replacing Y by U in equation (2.2)d and using the equation (1.1)e, we obtain (2.2)e. Replacing X by U in equation (2.2)e, we get (2.2)f. Equations (2.2)g and (2.2)h are obtained by replacing X by U in equation (2.1)a and (2.1)b. Barring Y in equation (2.2)b and using the equation (1.1)d, we get the equation (2.2)i.

Theorem (2.2). Let the connexion D and E be related by

$$E_X Y = P_1 D_X Y + P_2 \overline{D_X Y} + P_3 D_X \bar{Y} + P_4 \overline{D_X \bar{Y}} + P_5 \overline{D_X Y} + P_6 \overline{D_X Y} + P_7 \overline{D_X \bar{Y}} + P_8 \overline{D_X \bar{Y}}. \quad (2.3)$$

If D be general F-connexion then E is a general F-connexion, if

$$E_X Y = P_1 (D_X Y + \overline{D_X Y}) + P_2 (D_X Y + \overline{D_X Y}) + P_3 (D_X \bar{Y} + \overline{D_X \bar{Y}}) + P_4 (D_X \bar{Y} + \overline{D_X \bar{Y}}). \quad (2.4)$$

Proof. Barring Y throughout in equation (2.3) and using (1.1)a, (1.1)c, (2.2)b and (2.2)c, we get

$$E_X \bar{Y} = (E_X F)Y + \overline{E_X Y} = P_1 D_X \bar{Y} + P_2 \overline{D_X \bar{Y}} + P_3 (D_X Y - u(D_X Y)U) + P_4 (\overline{D_X Y} - u(\overline{D_X Y})U) + P_5 \overline{D_X \bar{Y}} + P_6 \overline{D_X \bar{Y}} + P_7 \overline{D_X \bar{Y}} + P_8 \overline{D_X \bar{Y}}. \quad (2.5)$$

Barring the equation (2.3) throughout and using the equation (1.1)a and (2.2)i, we get

$$\overline{E_X Y} = P_1 \overline{D_X Y} + P_2 \overline{D_X Y} + P_3 \overline{D_X \bar{Y}} + P_4 \overline{D_X \bar{Y}} + P_5 (D_X Y - u(D_X Y)U) + P_6 (\overline{D_X Y} - u(\overline{D_X Y})U) + P_7 \overline{D_X \bar{Y}} + P_8 \overline{D_X \bar{Y}}. \quad (2.6)$$

Subtracting the equation (2.6) from (2.5), we get

$$E_X \bar{Y} - \overline{E_X Y} = (E_X F)Y = (P_1 - P_7)(D_X \bar{Y} - \overline{D_X \bar{Y}}) + (P_2 - P_8)(D_X \bar{Y} + \overline{D_X \bar{Y}}) + (P_3 - P_5)(D_X Y - u(D_X Y)U - \overline{D_X Y}) + (P_4 - P_6)(D_X Y - u(D_X Y)U - \overline{D_X Y}). \quad (2.7)$$

Now $(E_X F)Y = 0$, if

$$P_1 = P_7, \quad P_2 = P_8, \quad P_3 = P_5, \quad P_4 = P_6. \quad (2.8)$$

Substituting from (2.8) in (2.3), we get the equation (2.4).

Corollary (2.1). For the general F-connexion D in M_n , equation (2.4) is equivalent to

$$E_X \bar{Y} = \overline{E_X Y} = P_1 (D_X \bar{Y} + \overline{D_X \bar{Y}}) + P_2 (D_X \bar{Y} + \overline{D_X \bar{Y}}) + P_3 (\overline{D_X Y} + D_X Y) + P_4 (\overline{D_X \bar{Y}} + D_X Y) - P_3 u(D_X Y)U - P_4 u(\overline{D_X Y})U \quad (2.9)$$

$$E_X Y = P_1 (D_X Y + \overline{D_X Y}) + P_2 (D_X Y + \overline{D_X Y}) + P_3 (D_X \bar{Y} + \overline{D_X \bar{Y}}) + P_4 (D_X \bar{Y} + \overline{D_X \bar{Y}}) - u(X)\{P_2 (D_U Y + \overline{D_U Y}) + P_4 (D_U \bar{Y} + \overline{D_U \bar{Y}})\}. \quad (2.10)$$

Proof. Barring Y in equation (2.4) and barring (2.4) throughout and using the equations (1.1)a, (1.1)c and (2.2)i in the resulting equations, we get the equation (2.9). Barring X in (2.4) and using (1.1)a, (1.1)c, (2.2)b and (2.2)i in the resulting equation, we get the equation (2.10).

3. O^* -F-Connexion or Quasi F-Connexion

A connexion D in M_n is called an O^* -F-connexion or Quasi F-connexion, if

$$(D_X F)Y + (D_X^* F)\bar{Y} = 0. \quad (3.1)$$

In view of (1.1)a, we have

$$D_X \bar{Y} - \overline{D_X Y} + D_X^* Y - D_X^* (u(Y)U) - \overline{D_X^* Y} = 0, \quad (3.2)a$$

equivalently

$$D_x \bar{Y} - \overline{D_x Y} + D_x Y - \{(D_x u)Y + u(D_x Y)\}U - u(Y)D_x U - \overline{D_x Y} = 0 \quad (3.2)b$$

Barring Y in equation (3.2)a and using (1.1)d, we

$$\text{get } u(D_x Y)U - D_x u(Y)U + u(D_x \bar{Y})U = 0 \quad (3.2)c$$

Barring (3.2)c throughout and using (1.1)c, we get

$$D_x u(Y)U = 0 = D_x u(Y)U, \quad (3.2)d$$

Substituting $X = U$ in (3.2)d and using (1.1)c, we get

$$D_U u(Y)U = 0. \quad (3.2)e$$

Theorem (3.1). For an O^* -F-connexion in M_n , we have

$$-D_x \bar{U} - \{(D_x u)(U) + u(D_x U)\}U = 0 \quad (3.3)a$$

$$-D_x U + u(D_x U)U = 0 = -D_x U + u(D_x U)U, \quad (3.3)b$$

$$D_x \bar{U} = 0 = \overline{D_x U}, \quad (3.3)c$$

$$(D_x u)(U) + u(D_x U) = 0, \quad (3.3)d$$

$$D_U \bar{Y} = \overline{D_U Y}, \quad (3.3)e$$

$$D_U \bar{U} = 0, \quad (3.3)$$

$$D_U U - u(D_U U)U = 0, \quad (3.3)g$$

$$u(D_U \bar{Y}) = 0, \quad (3.3)h$$

$$(D_U u)(Y) + u(Y)u(D_U U) = 0 \quad (3.3)i$$

$$(D_U u)(U) + u(D_U U) = 0, \quad (3.3)j$$

$$(D_U u)(\bar{Y}) = 0, \quad (3.3)k$$

$$u(D_x \bar{Y}) - (D_x u)(Y) - u(Y)u(D_x U) = 0, \quad (3.3)l$$

$$(D_x u)(\bar{Y}) + u(D_x \bar{Y}) = 0 = (D_x u)(\bar{Y}) + u(D_x \bar{Y}), \quad (3.3)m$$

$$D_x \bar{Y} - D_x Y + u(D_x Y)U + \overline{D_x Y} - D_x \bar{Y} + u(D_x \bar{Y})U = 0, \quad (3.3)n$$

$$D_U \bar{Y} - D_U Y + u(D_U Y)U = 0, \quad (3.3)o$$

$$(D_U u)(Y)U + u(Y)D_U U = 0. \quad (3.3)p$$

Proof. Putting $Y = U$ in (3.2) and using

(1.1)c and (1.1)e in the resulting equation, we get the equation (3.3)a. Barring the equation (3.3) throughout and using (1.1)a and (1.1)c, we obtain the equation (3.3)b. Barring (3.3)b and using (1.1)c, we obtain (3.3)c. Using the equation (3.3)c in (3.3)a then we get the equation (3.3)d. Putting $X = U$ in (3.2)a and

using (1.1)c, we get the equation (3.3)e. Putting $Y = U$ in (3.3)e and using (1.1)c, we

get (3.3)f. Barring (3.3)f and using (1.1)a, we

get the equation (3.3)g. Applying u in equation (3.3)e and using (1.1)d, we get (3.3)h. Barring Y in (3.3)h and using (1.1)a, (1.1) e, we get (3.3)i. Putting $Y = U$ in (3.3)i

and using (1.1)e, we get (3.3)j. Barring Y in (3.3)i and using (1.1)d, we get (3.3)k.

Applying u on (3.2)b and using (1.1)d, we get (3.3)l. Differentiating the equation (1.1)d with respect to X and \bar{X} , we get the equation (3.3)m. Barring the equation (3.2)b throughout and using the equations (1.1)a, (1.1)c and (3.3)c, we get (3.3)n. Putting $X = U$ in (3.3)n and using (1.1)e, we get the

equation (3.3)o. Barring Y in equation (3.3)e and using (1.1)a, (3.3)o in the resulting equation then we get the equation (3.3)p.

Theorem (3.2). Let D be an O^* -F-connexion in M_n and E is an arbitrary connexion in M_n , then E is an O^* -F-connexion in M_n , if

$E_x Y = (P_6 + P_7 + P_4)D_x Y + (P_5 + P_8 - P_3)D_x \bar{Y} + P_3 D_x \bar{Y} + P_4 \overline{D_x \bar{Y}} + P_5 \overline{D_x \bar{Y}}$

$+ P_6 \overline{D_x \bar{Y}} + P_7 \overline{D_x \bar{Y}} + P_8 \overline{D_x \bar{Y}}.$ (3.4)

Proof. Barring the equation (3.3) throughout and using the equation (1.1)a, we get

$$\begin{aligned} \overline{E_X Y} &= P_1 \overline{D_X Y} + P_2 \overline{D_X Y} + P_3 \overline{D_X Y} + P_4 \overline{D_X Y} + P_5 (D_X Y - u(D_X Y)U) \\ &+ P_6 (D_X Y - u(D_X Y)U) + P_7 (D_X Y - u(D_X Y)U) \\ &+ P_8 (D_X Y - u(D_X Y)U). \end{aligned} \quad (3.5)$$

Barring X, Y in equation (3.5) and using (1.1)a, (1.1)c, (3.2)d and (3.2)e, we get

$$\begin{aligned} \overline{E_X Y} &= P_1 \overline{D_X Y} + P_2 (\overline{D_X Y} - u(X) \overline{D_U Y}) + P_3 \overline{D_X Y} + P_4 (\overline{D_X Y} - u(X) \overline{D_U Y}) \\ &+ P_5 (D_X Y - u(D_X Y)U) + P_6 (D_X Y - u(D_X Y)U) \\ &+ P_7 (D_X Y - u(D_X Y)U) + P_8 (D_X Y - u(D_X Y)U). \end{aligned} \quad (3.6)$$

Adding the equations (3.5) and (3.6), we get

$$\begin{aligned} \overline{E_X Y} + \overline{E_X Y} &= (P_1 + P_4) (\overline{D_X Y} + \overline{D_X Y}) + (P_2 + P_3) (\overline{D_X Y} + \overline{D_X Y}) \\ &+ (P_5 + P_8) (D_X Y + D_X Y - u(D_X Y)U - u(D_X Y)U) \\ &+ (P_6 + P_7) (D_X Y + D_X Y - u(D_X Y)U - u(D_X Y)U) \\ &- P_2 u(X) \overline{D_U Y} - P_4 u(X) \overline{D_U Y}. \end{aligned} \quad (3.7)$$

Barring Y in (3.3) and using (1.1)a, (1.1)c and (3.2)d, we get

$$\begin{aligned} \overline{E_X Y} &= (E_X F)Y + \overline{E_X Y} = P_1 \overline{D_X Y} + P_2 \overline{D_X Y} + P_3 (D_X Y - D_X u(Y)U) \\ &+ P_4 (D_X Y - D_X u(Y)U) + P_5 \overline{D_X Y} + P_6 \overline{D_X Y} \\ &+ P_7 \overline{D_X Y} + P_8 \overline{D_X Y}. \end{aligned} \quad (3.8)$$

Barring X, Y in (3.8) and using the equations (1.1)a, (1.1)c, (1.1)d, (3.2)d and (3.2)e we get

$$\begin{aligned} (E_X F)(Y) + \overline{E_X Y} &= P_1 (D_X Y - D_X u(Y)U) + P_2 (D_X Y - u(X)D_U Y - D_X u(Y)U + u(X)D_U u(Y)U) \\ &+ P_3 \overline{D_X Y} + P_4 \overline{D_X Y} - P_4 u(X) \overline{D_U Y} + P_5 \overline{D_X Y} + P_6 \overline{D_X Y} - P_6 u(X) \overline{D_U Y} \\ &+ P_7 \overline{D_X Y} + P_8 \overline{D_X Y} - P_8 u(X) \overline{D_U Y}. \end{aligned} \quad (3.9)$$

Adding the equations (3.8) and (3.9), we get

$$\begin{aligned} (E_X F)Y + (E_X F)Y + \overline{E_X Y} + \overline{E_X Y} &= (P_1 + P_4) (D_X Y + D_X Y - u(D_X Y)U - u(D_X Y)U) \\ &- P_2 u(X) \overline{D_U Y} \\ &+ (P_2 + P_3) (D_X Y + D_X Y - D_X u(Y)U) - P_4 u(X) \overline{D_U Y} \end{aligned}$$

$$\begin{aligned} &+ (P_5 + P_8) (\overline{D_X Y} + \overline{D_X Y}) + (P_6 + P_7) (\overline{D_X Y} + \overline{D_X Y}) \\ &- P_6 u(X) \overline{D_U Y} - P_8 u(X) \overline{D_U Y}. \end{aligned} \quad (3.10)$$

Using the equations (3.2)c and (3.3)e in equation (3.10), we get

$$\begin{aligned} (E_X F)Y + (E_X F)Y + \overline{E_X Y} + \overline{E_X Y} &= (P_1 + P_4) (D_X Y + D_X Y \\ &+ (D_X Y)U - u(D_X Y)U) - P_2 u(X) \overline{D_U Y} \end{aligned}$$

$$\begin{aligned} &+ (P_2 + P_3) (D_X Y + D_X Y - u(D_X Y)U - u(D_X Y)U) - P_4 u(X) \overline{D_U Y} \\ &+ (P_5 + P_8) (\overline{D_X Y} + \overline{D_X Y}) \\ &+ (P_6 + P_7) (\overline{D_X Y} + \overline{D_X Y}) - P_6 u(X) \overline{D_U Y} - P_8 u(X) \overline{D_U Y}. \end{aligned} \quad (3.11)$$

Since D is an O*-F-connexion in M_n then the necessary and sufficient condition that E is an O*-F-connexion in M_n is obtained by comparing the equations (3.7) and (3.11), we get

$$\begin{aligned} P_1 + P_4 &= P_6 + P_7 \quad \text{i.e.} \quad P_1 = P_6 + P_7 - P_4, \\ P_2 + P_3 &= P_5 + P_8 \quad \text{i.e.} \quad P_2 = P_5 + P_8 - P_3. \end{aligned}$$

Substituting from (3.12) in (2.3), we get the equation (3.4).

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