AN OPTIMAL BIVARIATE MIXED POLICY FOR A REPAIRABLE SYSTEM USING ALPHA SERIES PROCESS

Ms. N. Joshna[1], Prof. A. Mallikarjuna Reddy[2]
Research Scholar[1], Professor[2], Department of Mathematics, S.K.University, Anantapuramu, A.P, India.
jyothi700@rediffmail.com[1], prof.malliavutala@gmail.com[2]

Abstract- **This paper proposed an alternative alpha series repair mixed model based on preventive repair and failure repair. It assumes that the repair time and working time of the system form an increasing alpha series process and decreasing alpha series process respectively. Further we assumed that as soon as the system reliability reaches to predetermined reliability 'R' a preventive repair is employed to restore the system for better working condition. Under these assumptions using geometric process we derived an expression for the average cost rate under policy (R,N) in which the system is replaced whenever the number of failures reaches to 'N' or the reliability 'R' whichever occurs first. We determined an optimal replacement policy (R*,N*) such that the average cost rate is minimum. Finally, numerical results are provided to highlight the theoretical results.**

Key Words: **Replacement policy, renewal reward theorem, geometric process, alpha series process, and long-run average cost per unit time.**

1. INTRODUCTION

Most repair replacement models assume that a failure system after repair will yield a function system which is **"as good as new"** and the repair times are neglected, so that the successive operating times forms a "**renewal process".** These types of models may be called as "**perfect repair models"**.

Barlow and Hunter [1] studied a minimal repair model in which a minimal repair does not change the age of the system. Thereafter an imperfect repair model was developed by Brown and Proschan [2] under which a repair with probability **'p'** as perfect repair and with probability **1-p** as minimal repair. Many others worked in this direction and developed corresponding optimal replacement polices by Lam Yeh[3],Park [6], Stadje and Zuckerman [7], Stanely [8].

In general, for a deteriorating system, it is reasonable to assume that the successive working times are stochastically decreasing while the consecutive repair times after failures are stochastically increasing, due to the ageing and accumulated wearing in many systems. To model such simple repairable deteriorating system Lam [4,5] first introduced a geometric process repair model under the assumptions that the system after repair is **'not as good as new.'** Under these assumptions, Wang[11] considered two kinds of replacement polices -one is based on the working age 'T' of the system and other is based on the number of failures 'N' of the system. Later Zhang [12] developed a bivariate replacement policy (T, N) to generalize Lam's work.

All these research works discussed above are related to one component repairable system. However, in many practical applications, the standby techniques are usually used for improving the reliability or raising the availability of the system. Zhang [14] applied the geometric process repair model to a single cold standby repairable system with one repairman and studied a replacement policy 'N' and corresponding optimal replacement policy 'N^*'is determined such that the long-run-average cost per unit time is minimum. Later, Zhang applied the geometric process repair model to a two unit cold standby repairable system with one repairman and studied a replacement policy 'N'. later on Wang and Zhang [2009] presented a bivariate mixed policy for a simple repairable system based preventive repair and failure repair by assuming that the repair time and working time of the system form an increasing geometric process and decreasing geometric process respectively. Further we assumed that as soon as the system reliability reaches to predetermined reliability 'R' a preventive repair is employed to restore the system for better working condition. Under these assumptions using geometric process they derived an expression for the average cost rate under policy (R,N) in which the system is replaced whenever the number of failures reaches to 'N' or the reliability 'R' whichever occurs first. We determined an optimal replacement policy (R*,N*) such that the average cost rate is minimum. Finally, numerical results are provided to highlight the theoretical results.

Zhang Y.L [13] studied some important properties of monotone processes and proved that alpha series processes is more appropriate to model the up times. On this understanding, in this chapter we have proposed to develop alpha series processes maintenance model and obtained an optimal replacement policy 'N*'

Based on this understanding, in this study we proposed an alternative alpha series repair mixed model based on preventive repair and failure repair.It assumes that the repair time and working time of the system form an increasing alpha series process and decreasing alpha series process respectively. Further we assumed that as soon as the system reliability reaches to predetermined reliability 'R' a preventive repair is employed to restore the system to better working condition. Under these assumptions using geometric process they derived an expression for the average cost rate under policy (R,N) in which the system is replaced whenever the number of failures reaches to 'N' or the reliability 'R' whichever occurs first. We determined an optimal replacement policy (R*,N*) such that the average cost rate is minimum. Finally, numerical results are provided to highlight the theoretical results.

2. THE MODEL

The following simple repairable systems with preventive repair and failure repair are considered by making the following assumptions:

1. In the beginning a new system is installed.
2. Assume that the system will be replaced by an identical one whenever it fails completely and replacement time is negligible.
3. Assume that the time interval between $(m-1)^{th}$ replacement and m^{th} replacement of the system is called m^{th} renewal cycle.
4. Assume that the time interval between $(n-1)^{th}$ failure repair of the system and n^{th} failure repair of the system is called n^{th} renewal cycle.
5. Assume that the preventive repair is employed once whenever the reliability of the system reaches to the critical reliability 'R'. The system after preventive repair is "as good as new" in the same repair cycle.
6. Assume that the failure repair of the system form an alpha series processes.
7. Let ψ_n, X_n, Y_n and ξ_n be respectively the working time with preventive repair, the working time without preventive repair, the repair time and total preventive repair time of the system ofn^{th}renewal cycle, n=1,2,3,……. .
8. Let the distribution function of working time and repair time of the systems be $F_n(n^{\alpha} t), G_n(n^{\beta_1} t)$ respectively. Where α>0, 0<β<1,t>0.

9. Let υ_n, τ_n, and $Z_n^{(j)}$ be respectively the number of preventive repairs, the length of working interval time between two preventive repairs, and j^{th} preventive repair of the system in the n^{th} cycle,(j=1,2,3,…, υ_n.). It can be observe that $X_n^{(m)} = \tau_n$ for m=1,2,… υ_n-1, while $X_n^{(\upsilon_n)} < \tau_n$.
10. Assume that $Z_n^{(j)}$,j=1,2,3,…υ_n are independent and identically distributed with distribution function $H_n(n^{\beta_2} t)$,where $0 < \beta_2 < 1$.
11. Let the total working time and total preventive repair time of the system in the n^{th} cycle are respectively
    Total working time = (working time with preventive repair+ working time without preventive repair).
    $\varphi_n = \psi_n + X_n^{(\upsilon_n)}$
    $\varphi_n = \upsilon_n \tau_n + X_n^{(\upsilon_n)}$, where n=1,2,3…. (2.1)
12. Assume that preventive repair cost rate is 'Cp', the failure repair cost rate is 'C_f', the working reward rate is 'Cw', and the replacement cost is 'C'.

3. THE AVERAGE COST RATE UNDER POLICY (R,N)

A bivariate replacement policy (R,N) based on the critical reliability 'R' before preventive repair and failure repair number 'N' of the system is considered.The problem is to determine an optimal failure number 'N' at fixed Reliability 'R' such that the average cost rate is minimum.

Assume that 'τ_1' be the first replacement time of the system under policy (R,N).In general, { τ_n, n=1,2,3,…} will form a renewal process.

Let C(R,N) be the average cost rate under policy (R,N).According to the renewal reward theorem see Ross[ ], we have

$$C(R,N) = \frac{E[\text{cost in a renewal cycle}]}{E[\text{Length in a renewal cycle}]},$$

$$C(R,N) = \frac{E[\Psi]}{E[L]}. \qquad (3.1)$$

Where $$\Psi = C_p \sum_{n=1}^{N} \xi_n + C_f \sum_{n=t}^{N-1} Y_n + C - C_w \sum_{n=1}^{N} \varphi_n \qquad (3.2)$$

$$L = \sum_{n=1}^{N} \xi_n + \sum_{n=t}^{N-1} Y_n + \sum_{n=1}^{N} \varphi_n \qquad (3.3)$$

Now, we evaluate the expected cost and expected length of the renewal of the system as follows:

$$E\Psi = C_p \sum_{n=1}^{N} E\xi_n + C_f \sum_{n=t}^{N-1} EY_n + C - C_w \sum E\varphi_n \qquad (3.4)$$

$$EL = \sum_{n=1}^{N} E\xi_n + \sum_{n=t}^{N-1} EY_n + \sum E\varphi_n \qquad (3.5)$$

According to the assumptions of the model, the preventive repair is executed once as soon as the working length reaches 'τ_n' or the system reliability reaches 'R'.

Thus, R=1-F(τ_1)=1-$F_n(\tau_n)$=1-F(n^α), for n=1,2,3,…..

Since the working time of the system is stochastically continuous random variable and form a decreasing alpha series process, we have

$$\tau_n = \frac{\tau_1}{n^\alpha} \tag{3.6}$$

Because, the critical reliability 'R' is same in any repair cycle of the system'υ_n', having the Geometric Distribution for every non-integer 'n'.

$$P(\upsilon_n = k) = R^k (1-R), k = 0,1,2,..... \tag{3.7}$$

Clearly 'υ_n' is independent of cycle number 'n'.

According to the assumption 11 or equation (2.1) we have

$$E\varphi_n = E\psi_n + EX_n^{(\upsilon_n)} \tag{3.8}$$

Let $E\psi_n = E(\upsilon_n \tau_n) = E\upsilon_n E\tau_n = \sum_{k=0}^{\infty} k\, P(\upsilon_n = k)\; E\tau_n$

$$E\psi_n = \sum_{k=o}^{\infty} k R^k (1-R) = \frac{R}{1-R}\tau_n \tag{3.9}$$

Let $EX_n^{(\upsilon_n)} = E(X_n / X_n < \tau_n)$

$$EX_n^{(\upsilon_n)} = \frac{1}{Fn(\tau_n)} \int_0^{\tau_n} X\, dF_n(X) \tag{3.10}$$

$$EX_n^{(\upsilon_n)} = \frac{1}{1-R} \int_0^{\tau_n} x \frac{n^\alpha}{\lambda} \exp(-n^\alpha x / \lambda)\; dx$$

$$= \frac{\lambda}{n^\alpha} - \frac{1}{n^\alpha}\left[\tau_1 \exp(-\tau_1/\lambda) + \lambda \exp(-\tau_1/\lambda)\right] \tag{3.11}$$

From equations (3.9) ,(3.11) we have

$$E\varphi_n = \frac{R}{1-R}\frac{\tau_1}{n^\alpha} + \frac{\lambda}{n^\alpha} - \frac{1}{n^\alpha}\left[\tau_1 \exp(-\tau_1/\lambda) + \lambda \exp(-\tau_1/\lambda)\right] \tag{3.10}$$

Let $E\xi_n = E\sum_{j=1}^{\upsilon_n} Z_n^{(j)}$

$$E\xi_n = \frac{R}{1-R}\frac{\mu_2}{n^{\beta_2}} \tag{3.12}$$

Let $EY_n = \int_0^\infty y\, dG_n(n^{\beta_1} y) = \frac{\mu_1}{n^{\beta_1}}$ (3.13)

From equation (3.4) the expected cost is given by:

$$E\Psi = \Bigg(C_p \sum_{n=1}^{N} \frac{R}{1-R}\frac{\mu_2}{n^{\beta_2}} + C_f \sum_{n=t}^{N-1} \frac{\mu_1}{n^{\beta_2}} + C$$
$$- C_w \sum_{n=1}^{N} \frac{R}{1-R}\frac{\tau_1}{n^\alpha} + \frac{\lambda}{n^\alpha} - \frac{1}{n^\alpha}\left[\tau_1 \exp(-\tau_1/\lambda) + \lambda \exp(-\tau_1/\lambda)\right]\Bigg) \tag{3.14}$$

$$EL = \Bigg(\sum_{n=1}^{N} \frac{R}{1-R}\frac{\mu_2}{n^{\beta_2}} + \sum_{n=t}^{N-1} \frac{\mu_1}{n^{\beta_2}} +$$
$$\sum_{n=1}^{N} \frac{R}{1-R}\frac{\tau_1}{n^\alpha} + \frac{\lambda}{n^\alpha} - \frac{1}{n^\alpha}\left[\tau_1 \exp(-\tau_1/\lambda) + \lambda \exp(-\tau_1/\lambda)\right]\Bigg) \tag{3.15}$$

Let $l_1 = \sum_{n=1}^{N} \frac{R}{1-R} \frac{\tau_1}{n^{\alpha}} + \frac{\lambda}{n^{\alpha}} - \frac{1}{n^{\alpha}} \left[\tau_1 \exp(-\tau_1 / \lambda) + \lambda \exp(-\tau_1 / \lambda) \right]$

$l_2 = \sum_{n=t}^{N-1} \frac{\mu_1}{n^{\beta_2}}$, and $l_3 = \sum_{n=1}^{N} \frac{R}{1-R} \frac{\mu_2}{n^{\beta_2}}$ then C(R,N) is given by

$$C(R,N) = \frac{C_p\, l_3 + C_f\, l_2 + C - C_w\, l_1}{l_1 + l_2 + l_3} \quad (3.16)$$

4. NUMERICAL RESULTS

α=0.315,β$_1$=-0.85,β$_2$=-0.65,τ-100,R=0.95,μ1=150, μ2=30,λ=30,cf=10,cw=20 cp=10 and C=710000

Table:4.1 The average cost rate of values against ‘N'

N	C(N)	N	C(N)
1	-7.64391	13	-8.14273
2	-9.85608	14	-7.66876
3	-10.7043	15	-7.18688
4	-10.9955	16	-6.69907
5	-11.0039	17	-6.20685
6	**-10.849**	18	-5.71144
7	-10.5917	19	-5.21383
8	-10.2663	20	-4.71481
9	-9.89365	21	-4.21503
10	-9.48716	22	-3.71503
11	-9.05591	23	-3.21525
12	-8.60624	24	-2.71606

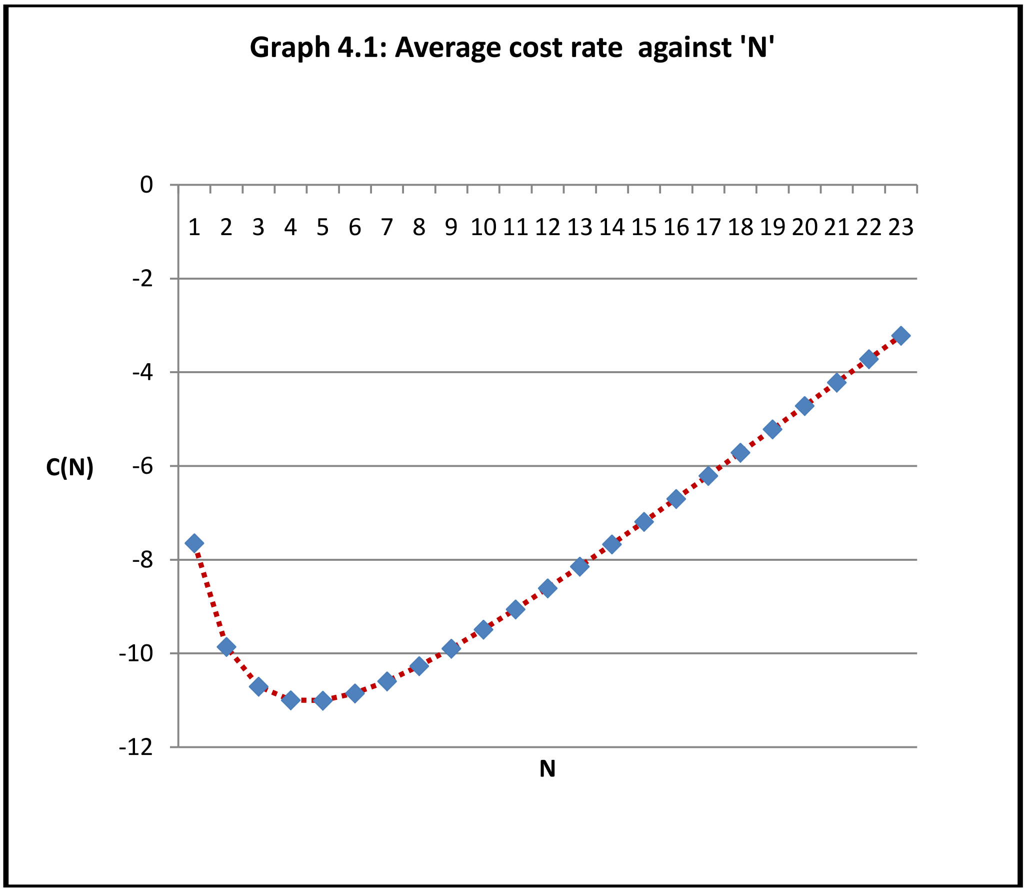


Table:4.2The average cost rate values against 'N'

α=0.315,β_1=-0.75,β_2=-0.55,τ-100,R=0.95,μ1=150, μ2=30,λ=30,cf=10,cw=20 cp=1and C=710000

N	C(N)	N	C(N)
1	-7.73007	13	-9.88209
2	-10.0466	14	-9.56284
3	-11.0113	15	-9.23645
4	-11.4276	16	-8.90469
5	-11.5676	17	-8.56894
6	**-11.5496**	18	-8.23029
7	-11.4333	19	-7.88958
8	-11.2521	20	-7.5475
9	-11.0264	21	-7.20459
10	-10.7692	22	-6.8613
11	-10.489	23	-6.51799
12	-10.1918	24	-6.17498

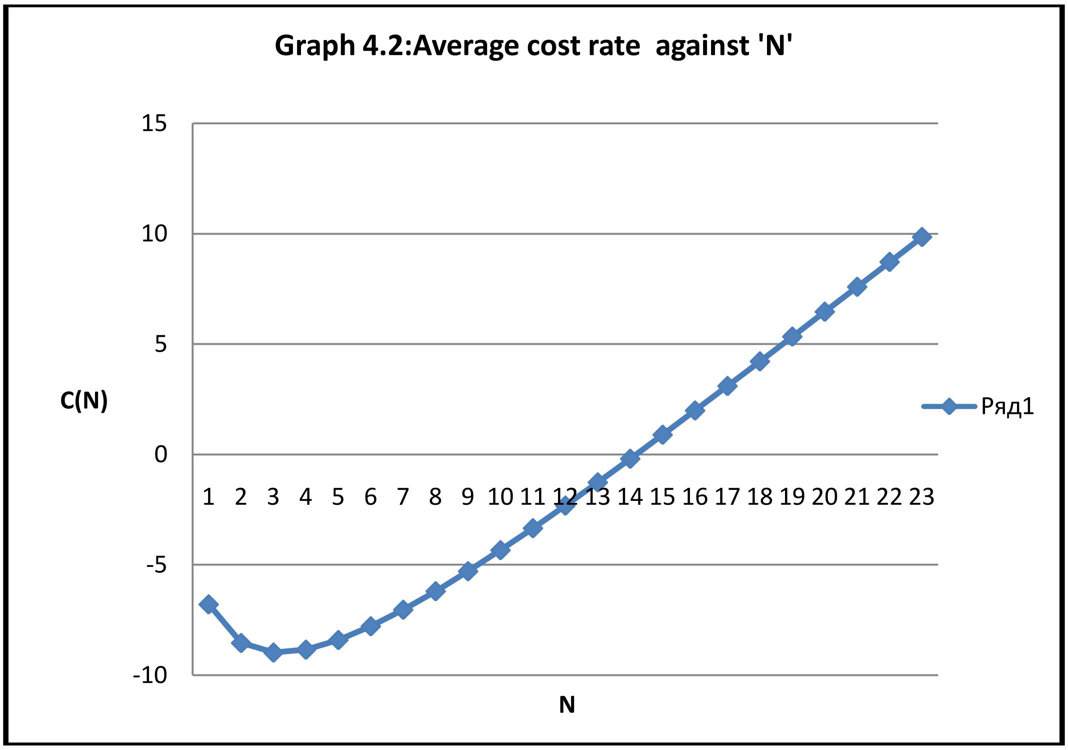


Table:4.3The average cost rate values against 'N'

α=0.315,β_1=-0.95,β_2=-0.75,τ-100,R=0.95,μ1=150, μ2=30,λ=30,cf=10,cw=20 cp=1and C=710000

N	C(N)	N	C(N)
1	-7.55157	13	-5.99986
2	-9.64647	14	-5.32455
3	-10.3596	15	-4.63967
4	-10.5022	16	-3.94752
5	**-10.3513**	17	-3.24993
6	-10.0285	18	-2.54839
7	-9.59618	19	-1.84413
8	-9.08971	20	-1.13815
9	-8.53098	21	-0.43131
10	-7.9342	22	0.275678
11	-7.30913	23	0.982205
12	-6.66266	24	1.687747

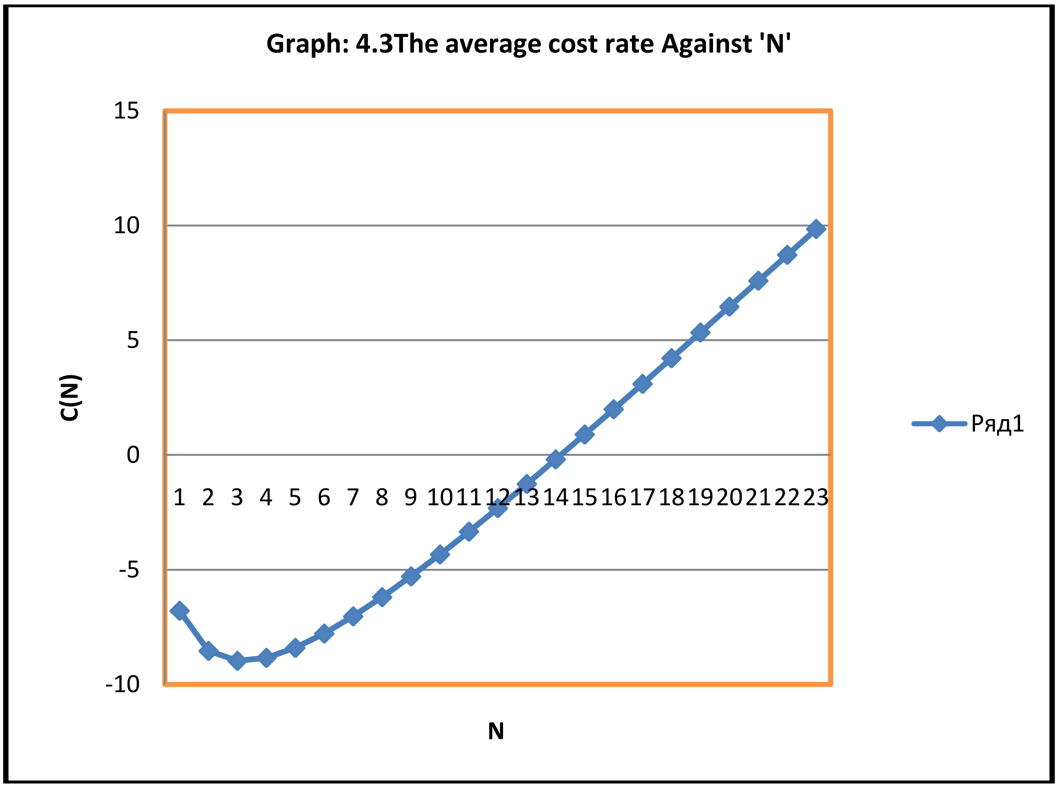


Table:4.4The average cost rate values against 'N'

α=0.415,β_1=-0.95,β_2=-0.75,τ-100,R=0.95,μ_1=150, μ_2=30,λ=30,C_f=10,C_w=20 cp=1and C=710000

N	C(N)	N	C(N)
1	-7.16996	13	-3.74
2	-9.09651	14	-2.87991
3	-9.67559	15	-2.00735
4	-9.68757	16	-1.12507
5	**-9.40142**	17	-0.2353
6	-8.93655	18	0.660099
7	-8.35494	19	1.559563
8	-7.69238	20	2.46177
9	-6.97125	21	3.365586
10	-6.20641	22	4.270029
11	-5.40823	23	5.174246
12	-4.58422	24	6.07749

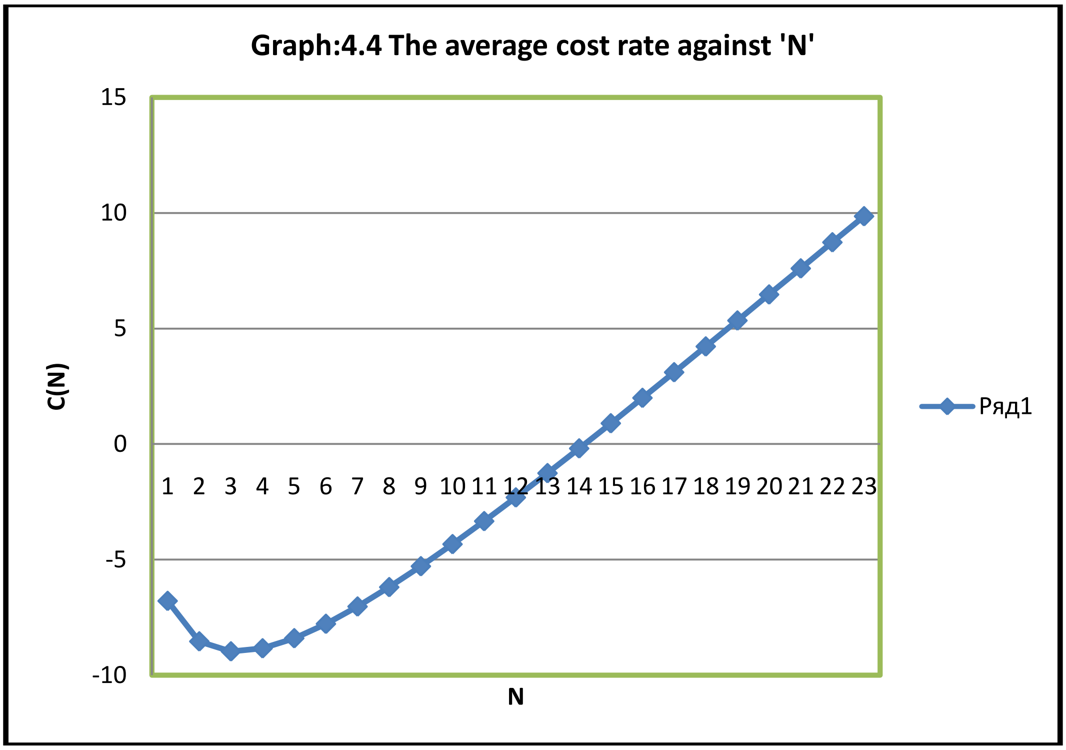
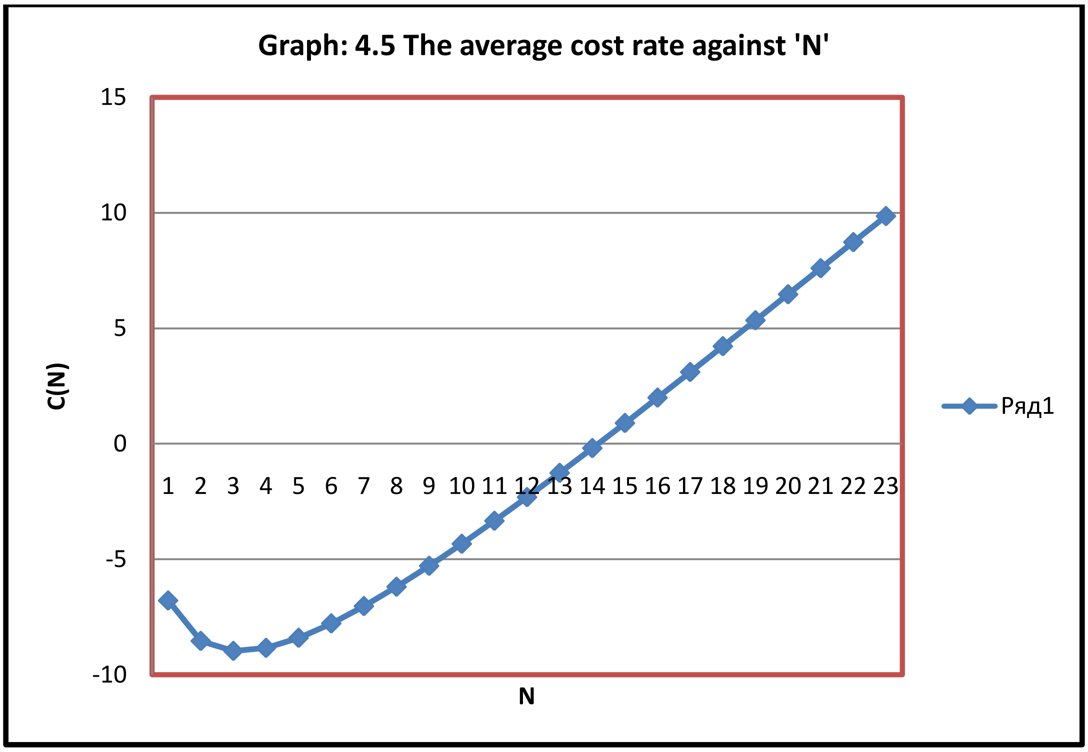


Table:4.5 The average cost rate values against ‘N’

α=0.515,β_1=-0.95,β_2=-0.75,τ-100,R=0.95,μ1=150, μ2=30,λ=30 cf=10,cw=20,cp=1and C=710000

N	C(N)	N	C(N)
1	-6.79217	13	-1.26469
2	-8.54166	14	-0.19381
3	**-8.9752**	15	0.892974
4	-8.8433	16	1.992361
5	-8.407	17	3.101579
6	-7.7835	18	4.218278
7	-7.03449	19	5.340442
8	-6.19617	20	6.466326
9	-5.29159	21	7.594404
10	-4.33636	22	8.723334
11	-3.34162	23	9.851929
12	-2.31562	24	10.97913



5. CONCLUSION

i. From the table 4.1 and graph 4.1, it is examined that the long-run average cost per unit timeC(6) =**-10.849** is minimum for the given**α=0.315,β_1=-0.85,β_2=-0.65**. Thus, we should replace the system at the time of 6^{th} failure.

ii. From the table 4.2 and graph 4.2, it is observed that the long-run average cost per unit time C(6) =**-11.5496**is minimum for the given**α=0.315,β_1=-0.75,β_2=-0.55**. We should replace the system at the time of 6^{th} failure. Thus, from the above conclusion (i) it can be concluded that the long-run average cost per unit time decreases with increasing '**α**'.

iii. From the table 4.3 and graph 4.3, it is observed that the long-run average cost per unit time C(5) =**-10.3513**is minimum for the given**α=0.315,β_1=-0.95,β_2=-0.75**. We should replace the system at the time of 5^{th} failure.

iv. From the table 4.4 and graph 4.4, it is observed that the long-run average cost per unit time C(5) =**-9.40142**is minimum for the given**α=0.415,β_1=-0.95,β_2=-0.75**. We should replace the system at the time of 5^{th} failure. Thus the value of '**α**' and C(N) are negatively related. Therefore it is possible to reduce the long run average cost per unit time just by changing the parameter value.

v. From the table 4.5 and graph 4.5, it is observed that the long-run average cost per unit time C(3) =**-8.9752** minimum for the given**α=0.515,β_1=-0.95,β_2=-0.75**. We should replace the system at the time of 3^{rd}failure.

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