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ON INTUITIONISTIC FUZZY SUB-IMPLICATIVE IDEALS OF BCI-ALGEBRAS

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***Abstract-* The aim of this paper is to introduce the notion of intuitionistic fuzzy sub-implicative ideals in BCI- algebras and to investigate some of their related properties..**

***Keywords:* intuitionistic fuzzy sub-implicative ideal, intuitionistic fuzzy positive implicative idea , intuitionistic fuzzy p-ideal , intuitionistic fuzzy characteristic sub-implicative ideal .**
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1. INTRODUCTION

The notion of BCK/BCI-algebras was introduced by Imai and Iseki in 1966.In the same year Iseki introduced the notion of a BCI-algebras which is a generalization of a BCK-algebras. After the introduction of the concept of fuzzy sets by L.A. Zadeh [7],several researches were conducted on the generalization of the fuzzy sets. The idea of intuitionistic fuzzy set was first introduced by K.T.Atanassov [1,2], as a generalization of the notion of fuzzy set. In this paper using Atanassov's idea ,we establish the intuitionistic fuzzification of the concept of sub-implicative ideals in BCI-algebras and investigate some of their properties .

2. PRELIMINARIES

In this section we include some elementary definitions that are necessary for this paper.
By a BCI- algebra we mean an algebra $(X,*,0)$ of type (2,0) satisfying the following conditions:

(1) $((x * y) * (x * z)) * (z * y) = 0$,
(2) $(x * (x * y)) * y = 0$,
(3) $x * x = 0$,
(4) $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y, z \in X$.

In a BCI-algebra X, we can define a partial ordering " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.A BCI-algebra X is said to be implicative if $(x * (x * y) * (y * x) = y * (y * x)$ for all $x, y \in X$. A mapping $f: X \to Y$ of BCI-algebras is called a homomorphism if $f(x*y) = f(x) * f(y)$ for all $x, y \in X$.
In any BCI- algebra X ,the following hold :

(5) $((x * z) * (y * z)) * (x * y) = 0$,
(6) $x * (x * (x * y)) = x * y$,
(7) $0 * (x * y) = (0 * x) * (0 * y)$,
(8) $x * 0 = x$,
(9) $(x * y) * z = (x * z) * y$,
(10) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$, for all $x, y, z \in X$.

Example 2.1 The set X = { 0, 1, 2, 3 } with the following Cayley table is a BCI - algebra

$*$	0	1	2	3
0	0	0	0	3
1	1	0	0	3
2	2	2	0	3
3	3	3	3	0

Throughout this paper X always means a BCI-algebra without any specification.

Definition 2.2 An non-empty subset A of X is a positive implicative ideal of X if for all $x\in X$,

(1) $0 * x \in A$ implies $x \in A$

(2) $((x * z) * z) * (y * z) \in A$ and $y \in A$ imply $x * z \in A$.

Definition 2.3. An non empty set A in X is called a P-ideal if it satisfies for all $x, y, z \in X$,

(1) $0 \in A$,

(2) $(x * z) * (y * z) \in A$ and $y\in A$ imply $x\in A$.

Definition 2.4 [7]. Let X be a non-empty set. A fuzzy set μ in X is a function $\mu : X \to [0, 1]$.

Definition 2.5 [7]. Let μ be a fuzzy set in X. For $t \in [0,1]$,the set $\mu_t = \{ x \in X \mid \mu(x) \geq t\}$ is called a level subset of μ.

Definition 2.6 . A fuzzy set μ in X is called a fuzzy ideal of X if

(1) $\mu(0) \geq \mu(x)$,

(2) $\mu(x) \geq \min\{ \mu(x * y), \mu(y)\}$, for all $x, y \in X$.

For any elements x ,y of a BCI-algebra , $x^n * y$ denotes

$x * (\ldots * (x * (x * y))\ldots)$ in which x occurs n times.

Definition 2.7 [3]. A fuzzy set μ in X is called a fuzzy sub-implicative ideal of X if

(1) $\mu(0) \geq \mu(x)$,

(2) $\mu(y^2 * x) \geq \min\{ \mu(((x^2 * y) * (y * x)) * z), \mu(z)\}$, for all $x, y, z \in X$.

Definition 2.8 [2]. An intuitionistic fuzzy set (IFS) A in a non empty set X is an object having the form $A = \{ < x, \mu_A(x), \nu_A(x) > / x \in X \}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Notation: For the sake of simplicity, we shall use the symbol $A = < \mu_A, \nu_A >$ for the

IFS $A = \{ < x, \mu_A(x), \nu_A(x) > / x \in X \}$.

Definition 2.9 [2]. Let A be an intuitionistic fuzzy set of a set X. For each pair $< t, s > \in [0, 1]$, the set $A_{<t, s>} = \{ x \in X : \mu_A(x) \geq t \text{ and } \nu_A(x) \leq s \}$ is called the level subset of A .

Definition 2.10 [2]. Let A be an IFS in X and let $t \in [0, 1]$ .Then the sets $U(\mu_A ; t) = \{ x \in X : \mu_A(x) \geq t \}$ and $L(\nu_A , t) = \{ x \in X : \nu_A(x) \leq t \}$ are called a μ-level t -cut and ν- level t-cut of A , respectively.

3. INTUITIONISTIC FUZZY SUB-IMPLICATIVE IDEALS

Definition 3.1. An intuitionistic fuzzy set A in X is called an intuitionistic fuzzy sub-implicative ideal of X if it satisfies:

(1) $\mu_A(0) \geq \mu_A(x)$,
(2) $\nu_A(0) \leq \nu_A(x)$,
(3) $\mu_A(y^2 * x) \geq \min\{ \mu_A(((x^2 * y) * (y * x)) * z), \mu_A(z)\}$,
(4) $\nu_A(y^2 * x) \leq \max\{\nu_A(((x^2 * y) * (y * x)) * z), \nu_A(z)\}$ for all $x, y, z \in X$.

Example 3.2. Let X = { 0,1,2 } with the following Cayley table be a BCI algebra.

*	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

Let $A = < \mu_A, \nu_A >$ be an IFS in X defined by $\mu_A(0) = \mu_A(1) = 0.6$, $\mu_A(2) = 0.2$ and $\nu_A(0) = \nu_A(1) = 0.2$ and $\nu_A(2) = 0.6$. Then A is an intuitionistic fuzzy sub-implicative ideal of X.

Theorem 3.3. Let A be an intuitionistic fuzzy set in X satisfying $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$. If A is an intuitionistic fuzzy sub-implicative ideal of X , then A satisfies the following inequality
$\mu_A(y^2 * x) \geq \mu_A((x^2 * y) * (y * x))$ and $\nu_A(y^2 * x) \leq \nu_A((x^2 * y) * (y * x))$ for all $x, y \in X$.

Proof. Let A be an intuitionistic fuzzy ideal of X. Then

$$\mu_A(y^2 * x) \geq \min\{ \mu_A(((x^2 * y) * (y * x)) * z), \mu_A(z)\},$$

and

$$\nu_A(y^2 * x) \leq \max\{\nu_A(((x^2 * y) * (y * x)) * z), \nu_A(z)\}$$

Taking z = 0 we get

$$\mu_A(y^2 * x) \geq \min\{ \mu_A(((x^2 * y) * (y * x)) * 0), \mu_A(0)\}$$

$= \mu_A((x^2 * y) * (y * x)), \mu_A(0)\}$
$= \mu_A((x^2 * y) * (y * x))$

and

$\nu_A(y^2 * x) \leq \max\{\nu_A(((x^2 * y) * (y * x)) * 0), \nu_A(0)\}$
$= \nu_A((x^2 * y) * (y * x)), \nu_A(0)\}$
$= \nu_A((x^2 * y) * (y * x))$

Theorem3.4. Every intuitionistic fuzzy sub- implicative ideal of X is an intuitionistic fuzzy ideal.

Proof. Let A be an intuitionistic fuzzy sub-implicative ideal of X. Then

(i) $\mu_A(0) \geq \mu_A(x)$,
(ii) $\nu_A(0) \leq \nu_A(x)$,
(iii) $\mu_A(y^2 * x) \geq \min\{\mu_A(((x^2 * y) * (y * x)) * z), \mu_A(z)\}$,
(iv) $\nu_A(y^2 * x) \leq \max\{\nu_A(((x^2 * y) * (y * x)) * z), \nu_A(z)\}$ for all x ,y, z $\in$ X.

Putting y = x in (iii)and (iv),we get

$\mu_A(x) = \mu_A(x^2 * x)$
$\geq \min\{\{\mu_A(((x^2 * x) * (x * x)) * z), \mu_A(z)\}$
$= \min\{\{\mu_A(x * z), \mu_A(z)\}$

$\nu_A(x) = \nu_A(x^2 * x)$
$\leq \max\{\{\nu_A(((x^2 * x) * (x * x)) * z), \nu_A(z)\}$
$= \max\{\{\nu_A(x * z), \nu_A(z)\}$

for all x ,z $\in$ X.

Hence A is an intuitionistic fuzzy ideal.

Theorem 3.5. An intuitionistic fuzzy ideal of X may not be an intuitionistic fuzzy sub- implicative ideal.

Proof. Let X = { 0,a,b,c } with the following Cayley table be a BCI- algebra.

*	0	a	b	c
0	0	0	0	c
a	a	0	0	c
b	b	b	0	c
c	c	c	c	0

Let A= < μ_A, ν_A > be an IFS in X defined by
$\mu_A(0) = 0.7$ and $\mu_A(x) = 0.2$ for all $x \neq 0$ and $\nu_A(0) = 0.2$ and
$\nu_A(x) = 0.7$ for all $x \neq 0$.Then A is an intuitionistic fuzzy ideal of X. ,but it is not an intuitionistic fuzzy sub-implicative ideal of X because

$\mu_A(a^2 * b) < \min\{\mu_A(((b^2 * a) * (a * b)) * 0), \mu_A(0)\}$
$\nu_A(a^2 * b) > \max\{\nu_A(((b^2 * a) * (a * b)) * 0), \nu_A(0)\}$

Theorem 3.6. Every intuitionistic fuzzy ideal satisfying the condition

$\mu_A(y^2 * x) \geq \mu_A((x^2 * y) * (y * x))$ and $\nu_A(y^2 * x) \leq \nu_A((x^2 * y) * (y * x))$ is an intuitionistic fuzzy sub-implicative ideal of X**.**

Proof. Let A be an intuitionistic fuzzy ideal of X satisfying

$\mu_A(y^2 * x) \geq \mu_A((x^2 * y) * (y * x))$ and $\nu_A(y^2 * x) \leq \nu_A((x^2 * y) * (y * x))$

$\mu_A(y^2 * x) \geq \mu_A((x^2 * y) * (y * x))$

$\geq \min\{\mu_A(((x^2 * x) * (x * x)) * z), \mu_A(z)\}$

$\nu_A(y^2 * x) \leq \nu_A((x^2 * y) * (y * x))$

$\leq \max\{\nu_A(((x^2 * x) * (x * x)) * z), \nu_A(z)\}$

This completes the proof.

Definition 3.7 An intuitionistic fuzzy set A in X is called an intuitionistic fuzzy positive implicative ideal of X if

(i) $\mu_A(0) \geq \mu_A(x)$,

(ii) $\nu_A(0) \leq \nu_A(x)$,

(iii) $\mu_A(x * z) \geq \min\{\mu_A(((x * z) * z) * (y * z)), \mu_A(y)\}$,

(iv) $\nu_A(x * z) \leq \max\{\nu_A(((x * z) * z) * (y * z)), \nu_A(y)\}$ for all x ,y, z $\in$X.

Example 3.8. Let X = { 0,a,b,c } with the following Cayley table be a BCI- algebra.

*	0	a	b	c
0	0	0	0	c
a	a	0	0	c
b	b	b	0	c
c	c	c	c	0

Let A = < μ_A, ν_A > be an IFS in X defined by

$\mu_A(0) = 0.7$ and $\mu_A(x) = 0.2$ for all $x \neq 0$ and $\nu_A(0) = 0.2$ and

$\nu_A(x) = 0.7$ for all $x \neq 0$.Then A is an intuitionistic fuzzy positive implicative ideal of X.

Theorem 3.9. Every intuitionistic fuzzy sub-implicative ideal is an intuitionistic fuzzy positive implicative ideal.

Proof. Let A be an intuitionistic fuzzy sub-implicative ideal of X. Then A is an intuitionistic fuzzy ideal of X. From Theorem 3.3

$\mu_A(b^2 * a) \geq \mu_A((a^2 * b) * (b * a))$ and $\nu_A(b^2 * a) \leq \nu_A((b^2 * a) * (b * a))$ for all a ,b$\in$X.

Substituting x*y for a and x for b we have

$\mu_A(x * y) = \mu_A(x * (x * (x * y)))$

$= \mu_A(b^2 * a)$

$\geq \mu_A((a^2 * b) * (b * a))$

$= \mu_A(((x * y) * ((x * y) * x))) * (x * (x * y)))$

$= \mu_A(((x * y) * (x * (x * y))) * ((x * y) * x))$

$= \mu_A(((x * (x * (x * y))) * y) * ((x * x) * y))$
$= \mu_A(((x * y) * y) * (0 * y))$

$\nu_A(x * y) = \nu_A(x * (x * (x * y)))$
$= \nu_A(b^2 * a)$
$\leq \nu_A((a^2 * b) * (b * a))$
$= \nu_A(((x * y) * ((x * y) * x))) * (x * (x * y)))$
$= \nu_A(((x * y) * (x * (x * y))) * ((x * y) * x))$
$= \nu_A(((x * (x * (x * y))) * y) * ((x * x) * y))$
$= \nu_A(((x * y) * y) * (0 * y))$

Hence A is an intuitionistic fuzzy positive implicative ideal of X.

Definition3.10 An intuitionistic fuzzy set A in X is called an intuitionistic fuzzy p-ideal of X if (i) $\mu_A(0) \geq \mu_A(x)$,
(ii) $\nu_A(0) \leq \nu_A(x)$,
(iii) $\mu_A(x) \geq \min\{\mu_A((x * z) * (y * z)), \mu_A(y)\}$,
(iv) $\nu_A(x) \leq \max\{\nu_A((x * z) * (y * z)), \nu_A(y)\}$ for all x ,y, z $\in$X.

Theorem 3.11 . An intuitionistic fuzzy p- ideal of X is an intuitionistic fuzzy sub-implicative ideal of X, but the converse does not hold.

Proof. Suppose that A is an intuitionistic fuzzy p- ideal of X . Then it is an intuitionistic fuzzy ideal of X. Note that

$(0^2 * (y^2 * x)) * (x^2 * y) * (y * x)) = (0 * ((x^2 * y) * (y * x))) * (0 * (y^2 * x))$
$= ((0 * ((x^2 * y)) * (0 * (y * x))) * ((0 * y) * (0 * (y * x)))$
$= (((0 * x) * (0 * ((x * y))) * (0 * (y * x))) * ((0 * y) * (0 * (y * x)))$
$\leq (((0 * x) * (0 * ((x * y))) * (0 * y)$
$= ((0 * x) * (0 * y)) * (0 * (x * y))$
$= 0$

Since in an intuitionistic fuzzy ideal A = < μ_A, ν_A>, μ_A is order reversing and
and ν_A is order preserving

$\mu_A(y^2 * x) \geq \mu_A(0^2 * (y^2 * x))$
$\geq \min\{\mu_A((0^2 * (y^2 * x)) * ((x^2 * y) * (y * x))), \mu_A((x^2 * y) * (y * x))\}$
$\geq \min\{\mu_A(0), \mu_A((x^2 * y) * (y * x))\}$
$= \mu_A((x^2 * y) * (y * x))$

$\nu_A(y^2 * x) \leq \nu_A(0^2 * (y^2 * x))$
$\leq \max\{\nu_A((0^2 * (y^2 * x)) * ((x^2 * y) * (y * x))), \nu_A((x^2 * y) * (y * x))\}$
$\leq \max\{\nu_A(0), \nu_A((x^2 * y) * (y * x))\}$
$= \nu_A((x^2 * y) * (y * x))$

Hence A is an intuitionistic fuzzy sub-implicative ideal.

Example 3.12. Let X = { 0,a,1,2,3 } with the following Cayley table be a BCI- algebra.

*	0	a	1	2	3
0	0	0	3	2	1
a	a	0	3	2	1
1	1	1	0	3	2
2	2	2	1	0	3
3	3	3	2	1	0

Let A= < μ_{A}, ν_A > be an IFS in X defined by
$\mu_A(0) = 0.7$, $\mu_A(a) = 0.5$ $\mu_A(1) = \mu_A(2) = \mu_A(3) = 0.2$ and
$\nu_A(0) = 0.2$, $\nu_A(a) = 0.5$,$\nu_A(1) = \nu_A(2) = \nu_A(3) = 0.7$. Then A is an intuitionistic fuzzy ideal of X. in which the inequalities $\mu_A(y^2 * x) \geq \mu_A(y^2 * x) * (y * x))$ and $\nu_A(y^2 * x) \leq \nu_A(y^2 * x) * (y * x))$ hold for all x ,y $\in$X . A is an intuitionistic fuzzy sub-implicative ideal of X by Theorem 3.7.
But it is not an intuitionistic fuzzy p- ideal of X, since

$\mu_A(a) < \min\{ \mu_A((a * 1) * (0 * 1)) , \mu_A(0)\}$

$\nu_A(a) > \max\{\nu_A((a * 1) * (0 * 1)) , \nu_A(0)\}$

The proof is complete.

Theorem 3.13. For any intuitionistic fuzzy sub-implicative ideal of X, the set
$X_A=\{ x\in X \mid \mu_A(x) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(0) \}$ is a sub-implicative ideal.
Proof. Clearly $0\in X_A$. Let x ,y, z $\in$ X be such that $((x^2 * y) * (y * x)) * z \in X_A$ and $z\in X_A$. Then $\mu_A(y^2 * x) \geq \min\{ \mu_A(((x^2 * y) * (y * x)) * z) , \mu_A(z)\}= \mu_A(0)$and
$\nu_A(y^2 * x) \leq \max\{\nu_A(((x^2 * y) * (y * x)) * z) , \nu_A(z)\}= \nu_A(0)$ for all x ,y, z$\in$X.
which implies $\nu_A(y^2 * x) = \nu_A(0)$ and $\nu_A(y^2 * x) = \nu_A(0)$.That is , $y^2 * x \in X_A$
Therefore X_A is a sub-implicative ideal of X.

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