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SIX SIGMA SINGLE SAMPLING VARIABLES PLAN INDEXED BY MAPD

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Abstract: **This paper presents the designing methodology for the Six Sigma Single Sampling Variable Plans [SSSSVP (n_σ; k_σ)] indexed by entry parameter "Maximum allowable percent defective" (MAPD). Table yielding plan for given set of entry parameters namely p*, inflection Quality level and h*, the relative slope at p* have been furnished.**
Keywords: **Single sampling variables plan, Six Sigma, MAPD, P*, inflection Quality level and h*, relative slope.**

I. INTRODUCTION

Acceptance sampling have two broad categories of sampling plans, known as attributes plans and variables plans, have been developed to deal with both qualitative and quantitative data. If the items in a sample are simply classified as defective or acceptable on the basis of the qualitative characteristic is quantitative (for example, the length or weight of an item), then either an attributes or variables plan may be employed. The attributes plan is still applicable because quantitative data can be converted into qualitative data by judging an item(s) which may be rejectable otherwise. This approach does not, however, makes full use of information conveyed by the sample measurements. If the form of the underlying distribution of the quality characteristic is known, then statistics based on the measurements themselves can be used in a variables sampling plan. Variable plans are usually based on the assumption that the quality characteristics have a normal distribution. If the population standard deviation is known, then the sample mean contains the information required to estimate the proportion defective. The known-sigma sampling plans have been given by Liberman and Resnikoff (1955) and Owen (1967). The quality level known as Inflection quality level, introduced by Mayer (1967) and studied by Soundararajan (1975) is the quality corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level p* is measured by 'p', the point at which tangent of the OC curve at the inflection point cuts the proportion defective axis. This concept can be extended to Repetitive Group sampling (RGS) under variables plan was introduced by Senthilkumar (2011). Radhakrishnan and Sivakumaran (2008) have constructed six sigma quality level in single sampling attribute plans. Senthilkumar and Esha Raffie (2013) have studied six sigma quality level in single sampling variable plans. The resulting plan would be designated as SSSSVP (n_σ;k_σ) and would be applied under the following conditions of applications:

The conditions under which variables single sampling plan are to be applied in an industry are as follows

(i) The production is steady, so that results of past, present and future lots are broadly indicative of a continuous process.
(ii) Lots are submitted substantially in the order of their production.
(iii) Inspection is by variables, with the lot quality defined as the proportion defective.
(iv) In the manufacturing process, automatic machines is to be used with less man power.

II. ASSUMPTIONS

(i) The Quality Characteristic x has a normal distribution with a known or unknown standard deviation.
(ii) A Unit is Defective if $x > U$ or $x < L$, where U and L are the upper and lower specification limits respectively.
(iii) The purpose is to control the fraction defective p in large lots submitted for inspection.

III. OPERATING PROCEDURE

The operating procedure of six sigma single sampling variables plan is described below:

1. Take a random sample of size n, say $(x_1, x_2, \ldots, x_n)$.
2. Compute $v=(U-\overline{X})/\sigma$ or $v=(\overline{X}-L)/\sigma$ (Known Standard deviation), and

   Compute $v=(U-\overline{X})/s$ or $v=(\overline{X}-L)/s$ (Unknown Standard deviation),

   where $s=[\sum(x_i-\overline{x})^2/(n-1)]^{1/2}$

   $\overline{X}=\sum x/n$
3. When $v \geq k$, the lot is accepted; when $v < k$, the lot is rejected (see Duncan, 1986).

For this plan, the probability of acceptance $P_a(p)$ of a lot is given by

$$P_a(p) = F(W) \quad \textbf{(1)}$$

where $w = (v-k)\sqrt{n}$ **(2)**

From the assumption of single sampling variable plans stated the probability of acceptance $P_a(p)$ of a lot is given by

$$F(y)=\int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz \qquad \textbf{(3)}$$

The quality level corresponding to the inflection point is denoted by p_* and the p_* of SSSSVP $(n_\sigma;k_\sigma)$ is the value of p such that

$$\frac{d^2P_a(p)}{dp^2}=0 \qquad \textbf{(4)}$$

where $P_a(p)$ is the proportion of the lots expected to be accepted while applying SSSSVP $(n_\sigma; k_\sigma)$. The relative slope of the OC curve at $p=p_*$ is denoted by h^*. The h^* of SSSSVP $(n_\sigma;k_\sigma)$ is such that

$$h^* = -\left[\frac{p}{P_a(p)}\frac{dP_a(p)}{dp}\right] \text{ at } p=p^* \qquad \textbf{(5)}$$

in which

$$\frac{dP_a(p)}{dp}=$$

$$P_a'(p) = -\sqrt{n\exp\left(v^2-w^2\right)} \qquad \textbf{(6)}$$

and

$$\frac{d^2Pa(p)}{dp^2}=$$

$$P_a'' = \sqrt{n_\sigma}\exp((1/2)(v^2-w^2))\sqrt{2\pi}\exp(v^2/2)\left[-(v-k)n+v\right] \qquad \textbf{(7)}$$

Designing SSSSVP (n, k) with known standard deviation for given h* and p*

Example 1

Table 1 can be used to determine SSSSVP (n, k) for specified values of h* and p*. For example, if it is desired to have a SSSSVP (n, k) for given p* = 0.00001 and h*= 15, Table 1 gives n = 109, and k = 5.215.

Designing SSSSVP (n, k) with unknown standard deviation for given h* and p*

Example 1

Table 1 can be used to determine SSSSVP (n, k) for specified values of h* and p*. For example, if it is desired to have a SSSSVP (n, k) for given p* = 0.00001 and h*= 15, Table 1 gives n = 1591, and k = 5.216.

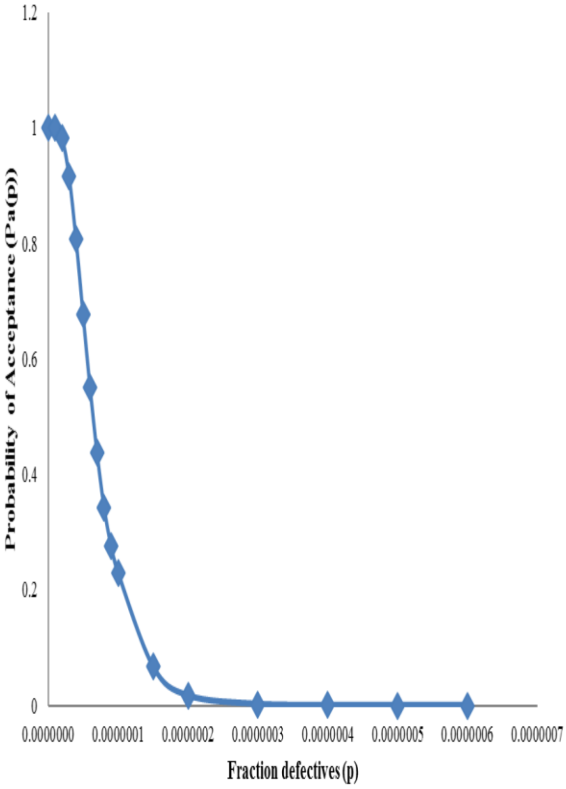

Figure 1. OC Curves of SSSSVP with n = 101, k = 5.218, and h*= 16

IV. CONSTRUCTION OF TABLE 1

When the basic assumptions with regard to variable sampling are satisfied, the fraction defective in a lot will be

$p=1-F(v) = F(-v)$ with $v=(U-\mu)/\sigma$

and $F(y)= \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$

Let $P_a(p_*)$ be the probability for acceptance and rejection respectively in a particular sample when $p = p_*$

For the given p_*, the value of v and w are obtained from approximation for the ordinate of the cumulative normal distribution.

Using iterative procedure equations (3) and (4) are solved for the given value of p_* ranging from 0.00001 to 0.0009 to get the values of n_σ and k_σ. By definition, the relative slope h_* at $p=p_*$ is given in equation (5). By substituting the value of n_σ, k_σ and p_* in the equation (5), h_* values are obtained and they are tabulated in Table 1.

A procedure for finding the parameters of unknown standard deviation method plan from known standard deviation method plan with parameter (n_s, k_s), where desired using Hamaker (1979) approximation as follows

$$n_S = n(1+\frac{k^2}{2}) \;\; and \;\; k_S = k\,\frac{(4n_S-4)}{(4n_S-5)}$$

Table 5.1 provides the values of n_σ, k_σ, n_s, and k_s and which satisfy equations (5.3) and (5.4).

V. CONCLUSION

It is concluded from the study that the sample size required for SSSSVP indexed through MAPD is less than that of the sample size of the SSSSVP indexed through AQL with more probability of acceptance and less inspection cost. This plan offer effectiveness and flexibility to the floor engineers and help them to decide their sampling plans on the floor itself and can take quick decisions to make the system very fast, effective and friendly.

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Table 1: Six sigma single sampling plan indexed by AQL and MAPD

p*	Methods	h*=15		h*=16		h*=17		h*=18	
		n	k	n	k	n	k	n	k
0.00001	σ	109	5.215	101	5.218	103	5.231	109	5.254
	s	1591	5.216	1476	5.219	1512	5.231	1613	5.255
0.00002	σ	74	5.210	79	5.213	85	5.226	91	5.249
	s	1078	5.211	1153	5.214	1246	5.227	1345	5.250
0.00003	σ	65	5.205	72	5.208	77	5.221	82	5.244
	s	945	5.206	1049	5.210	1126	5.222	1209	5.245
0.00004	σ	62	5.193	67	5.196	78	5.208	79	5.231
	s	898	5.194	971	5.197	1136	5.209	1160	5.233
0.00005	σ	61	5.180	66	5.183	70	5.195	74	5.218
	s	879	5.181	952	5.184	1015	5.196	1082	5.220
0.00006	σ	53	5.166	62	5.169	66	5.181	67	5.204
	s	760	5.167	890	5.170	952	5.182	974	5.206
0.00007	σ	53	5.154	57	5.157	63	5.169	67	5.192
	s	757	5.155	815	5.158	905	5.171	970	5.194
0.00008	σ	50	5.143	55	5.146	61	5.158	65	5.181
	s	711	5.144	783	5.147	872	5.160	938	5.183
0.00009	σ	48	5.131	53	5.134	57	5.146	60	5.169
	s	680	5.132	751	5.136	812	5.148	862	5.171
0.0001	σ	47	5.118	50	5.121	53	5.134	59	5.157
	s	663	5.120	706	5.123	751	5.135	844	5.158
0.0002	σ	40	5.106	42	5.109	45	5.122	48	5.145
	s	561	5.108	590	5.111	635	5.124	683	5.147
0.0003	σ	39	5.095	42	5.098	42	5.110	46	5.133
	s	545	5.097	588	5.100	590	5.112	652	5.135
0.0004	σ	37	5.083	38	5.086	39	5.098	40	5.121
	s	515	5.085	529	5.088	546	5.100	565	5.124
0.0005	σ	34	5.069	35	5.072	36	5.085	37	5.108
	s	471	5.072	485	5.075	501	5.087	520	5.110
0.0006	σ	32	5.046	33	5.049	34	5.062	35	5.085
	s	439	5.049	454	5.052	470	5.064	487	5.088
0.0007	σ	31	5.025	32	5.028	33	5.041	34	5.064
	s	422	5.028	437	5.031	452	5.043	470	5.067
0.0008	σ	29	5.010	30	5.013	31	5.025	32	5.049
	s	393	5.013	407	5.016	422	5.028	440	5.051
0.0009	σ	28	4.860	29	4.863	30	4.875	31	4.899
	s	359	4.863	372	4.866	387	4.878	403	4.902

Table 1 (continued…)

p*	Methods	h*=19		h*=20		h*=21		h*=22	
		n	k	n	k	n	k	n	k
0.00001	σ	124	5.277	130	5.301	137	5.304	144	5.317
	s	1851	5.278	1956	5.301	2064	5.304	2180	5.318
0.00002	σ	95	5.272	101	5.296	105	5.299	110	5.312
	s	1415	5.273	1517	5.296	1579	5.300	1662	5.313
0.00003	σ	86	5.267	91	5.291	96	5.294	101	5.307
	s	1279	5.268	1365	5.291	1441	5.295	1523	5.308
0.00004	σ	81	5.255	84	5.278	86	5.281	88	5.295
	s	1199	5.256	1254	5.279	1285	5.282	1321	5.296
0.00005	σ	78	5.242	82	5.265	83	5.268	87	5.282
	s	1150	5.243	1219	5.266	1235	5.269	1300	5.283
0.00006	σ	71	5.228	75	5.251	79	5.254	86	5.268
	s	1041	5.229	1109	5.252	1170	5.255	1279	5.269
0.00007	σ	70	5.216	72	5.239	76	5.242	79	5.256
	s	1022	5.217	1060	5.240	1120	5.243	1170	5.257
0.00008	σ	66	5.205	69	5.228	73	5.231	77	5.245
	s	960	5.206	1012	5.229	1072	5.233	1136	5.246
0.00009	σ	61	5.193	67	5.216	69	5.219	71	5.233
	s	883	5.194	978	5.217	1009	5.221	1043	5.234
0.0001	σ	60	5.180	63	5.204	66	5.207	69	5.220
	s	865	5.182	916	5.205	961	5.208	1009	5.221
0.0002	σ	49	5.168	55	5.192	60	5.195	61	5.208
	s	703	5.170	796	5.193	870	5.196	888	5.210
0.0003	σ	47	5.157	51	5.180	52	5.183	53	5.197
	s	672	5.159	735	5.182	751	5.185	769	5.198
0.0004	σ	43	5.145	46	5.168	47	5.171	51	5.185
	s	612	5.147	660	5.170	675	5.173	736	5.186
0.0005	σ	40	5.131	43	5.155	44	5.158	48	5.171
	s	567	5.133	614	5.157	629	5.160	690	5.173
0.0006	σ	38	5.108	41	5.132	42	5.135	46	5.148
	s	534	5.111	581	5.134	596	5.137	656	5.150
0.0007	σ	37	5.087	40	5.111	41	5.114	45	5.127
	s	516	5.090	562	5.113	577	5.116	636	5.129
0.0008	σ	35	5.072	38	5.095	39	5.098	43	5.112
	s	485	5.074	531	5.098	546	5.101	605	5.114
0.0009	σ	34	4.922	37	4.945	38	4.948	42	4.962
	s	446	4.925	489	4.948	503	4.951	559	4.964

Table 1 (continued…)

p*	Methods	h*=23		h*=24		h*=25	
		n	k	n	k	n	k
0.00001	σ	151	5.317	157	5.328	164	5.329
	s	2286	5.318	2385	5.328	2493	5.330
0.00002	σ	115	5.312	121	5.323	126	5.324
	s	1738	5.313	1835	5.323	1912	5.325
0.00003	σ	105	5.307	110	5.318	121	5.319
	s	1584	5.308	1665	5.319	1833	5.320
0.00004	σ	92	5.295	97	5.305	101	5.307
	s	1382	5.296	1462	5.306	1523	5.307
0.00005	σ	91	5.282	95	5.292	99	5.294
	s	1360	5.283	1425	5.293	1486	5.294
0.00006	σ	90	5.268	94	5.278	97	5.280
	s	1339	5.269	1403	5.279	1449	5.281
0.00007	σ	83	5.256	87	5.266	90	5.268
	s	1229	5.257	1293	5.267	1339	5.269
0.00008	σ	80	5.245	84	5.255	87	5.257
	s	1180	5.246	1244	5.256	1289	5.258
0.00009	σ	75	5.233	78	5.243	81	5.245
	s	1102	5.234	1150	5.244	1195	5.246
0.0001	σ	73	5.220	76	5.231	79	5.232
	s	1068	5.222	1116	5.232	1160	5.233
0.0002	σ	63	5.208	66	5.219	74	5.220
	s	918	5.210	965	5.220	1082	5.221
0.0003	σ	57	5.197	58	5.207	69	5.209
	s	827	5.199	844	5.209	1005	5.210
0.0004	σ	53	5.185	55	5.195	59	5.197
	s	765	5.187	797	5.197	856	5.198
0.0005	σ	50	5.171	52	5.182	56	5.183
	s	719	5.173	750	5.183	808	5.185
0.0006	σ	48	5.148	50	5.159	54	5.160
	s	684	5.150	715	5.161	773	5.162
0.0007	σ	47	5.127	49	5.138	53	5.139
	s	665	5.129	696	5.140	753	5.141
0.0008	σ	45	5.112	47	5.122	51	5.124
	s	633	5.114	664	5.124	720	5.125
0.0009	σ	44	4.962	46	4.972	50	4.974
	s	586	4.964	615	4.974	668	4.976