ECCENTRICITY AND CRITICAL PATH

R.Malathi [1], J.Kavitha [2],
[1]Assistant Professor, [2]M.Phil scholar
Dept of Mathematics, Scsvmv Univeristy, Enathur, Kanchipuram,Tamil Nadu 631561
[1]malathihema@yahoo.co.in, [2]jkavi16@gmail.com

***ABSTRACT*- Let G be a graph and V be vertex of a graph G. The eccentricity of the vertex V is the maximum distance from V to any vertex. i.e., e(v)=max{d(u,v):w in V(G)}. Diameter of a graph is defined as length of the shortest path between the most distanced nodes. In this paper we are going to discuss about, how critical path implies eccentricity. Critical path means longest sequence of activity in a project plan, which must be completed on time for the project to complete on due date. An activity on the critical path cannot be started until its predecessors activity is complete, if it is delayed for a day, the entire project will be delayed for a day. If an activity has zero slack,it is on the critical path.**

Key words: **Critical Path , eccentricity, Network Analysis.**

I INTRODUCTION:

The eccentricity E (v) of a vertex v is the greatest geodesic distance between v and any other vertex. It can be thought of as how far a node is from the node most distant from it in the graph. The **radius** r of a graph is the minimum eccentricity of any vertex or, in symbols, $r = \min_{v\in V} E(v)$

The **diameter** d of a graph is the maximum eccentricity of any vertex in the graph. That is, d it is the greatest distance between any pair of vertices or, alternatively, $d = \max_{v\in V} E(v)$. To find the diameter of a graph, first find the shortest path between each pair vertices. The greatest length of any of these paths is the diameter of the graph.

A **central vertex** in a graph of radius r is one whose eccentricity is r—that is, a vertex that achieves the radius or, equivalently, a vertex v such that E(v) = r.An activity is said to be critical if a delay in its cause a further delay in the completion date of the entire project. In other word the activity with zero total float are known as critical activities. The sequence of critical activities in a critical network is called critical path.The critical path is the longest path in the network from the starting event to ending event and defines the minimum time required to complete the project.

II DEFINITION

A tree is an undirected graph in which any two vertices are connected by exactly one path. In other words, any connected graph without simple cycles is a tree. A forest is adisjoint union of trees.

A **tree** is an undirected simple graph *G* that satisfies any of the following equivalent conditions:
G is connected and has no cycles.
G has no cycles, and a simple cycle is formed if any edge is added to *G*.
G is connected, but is not connected if any single edge is removed from *G*.
G is connected and the 3-vertex complete graph K_3 is not a minor of *G*.
Any two vertices in *G* can be connected by a unique simple path.
If *G* has finitely many vertices, say *n* of them, then the above statements are also equivalent to any of the following conditions:
G is connected and has $n - 1$ edges.
G has no simple cycles and has $n - 1$ edges.

Definition

In a graph a vertex is said to be pendant vertex, if it has a degree one.

Definition

The eccentricity of a vertex e(v) of a connected graph G is the number $\max_{u\epsilon V(G)} d(u,v)$.

Definition

The radius of a graph G is denoted by rad G and it is defined as rad G=$\min_{v\in V(G)} e(v)$.

Definition

The diameter of a graph G is denoted by dia G=$\max_{v\in V(G)} d(u,v)$ or diam G=$\max_{v\in V(G)} e(v)$.

III THEOREM

Every tree has either on e or two centers.

Proof:

The maximum distance from a given vertex v to any other vertex is denoted by, max d(v,v_i) occurs only when v_i is a pendant vertex. Let T be a tree and it has more than two vertices. Then it has two or more pendant vertices. Removal of all the pendant vertices from T uniformly reduces the eccentricities of the remaining vertices (vertices in T^{I}) by one. Hence the centers of T are also the centers of T^{I} . From T^{I} we deletingall pendant vertices and get another tree $T^{I\ I}$ .Continuing this process, we either get a vertex, which is a center of T, or an edge, whose end vertices are the two centers of T.

Theorem:-

For any connected graph G, rad (G) ≤ diam (G) ≤ 2 rad (G).

Proof:- By definition the inequality rad(G) ≤diam(G) is true, since the smallest eccentricity can not exceed the largest eccentricity.

Hence rad(G) ≤ diam (G) ………..(1)

To prove second inequality choose a vertices u and v in a graph G such that d(u,v)=diam(G). Let w be a central vertex of G i.e. e(w) = rad(G)

Since d is a metric onV(G).
d(u,v)≤d(u,w)+d(w,v)≤rad(G)+rad(G)=2 rad(G)
=>d(u,v)≤2 rad(G)
i.e. diam(G)≤2 rad(G) ………..(2)
Hence from (1) and (2) we get rad (G) ≤ diam (G) ≤ 2 rad (G).

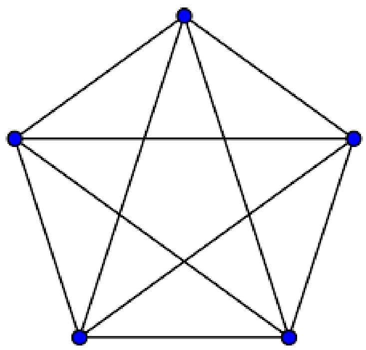
Definition: -

A vertex v is called an eccentric vertex of u, if e (u) = d (u, v); where u, v ∈ G, i.e. v is a vertex that is farthest from u. If v is an eccentric vertex of some vertex of G such a vertex v is an eccentric vertex of the graph G . In other words, a vertex v is an eccentric vertex of G, if v is farthest from some vertex of G.

Definition: -

A connected graph G has the same eccentricity & is there for a peripheral vertex, then certainly every vertex of G is an eccentric vertex.

A connected graph G is an eccentric graph if every vertex of G is an eccentric vertex.



Critical Path in Network Analysis

Basic scheduling computations

The notations used are

(i, j) = Activity with tail event i and head event j

E_i= Earliest occurrence time of event i

L_j= Latest allowable occurrence time of event

D_{ij}= Estimated completion time of activity (i, j)

$(Es)_{ij}$= Earliest starting time of activity (i, j)

$(Ef)_{ij}$= Earliest finishing time of activity (i, j)

$(Ls)_{ij}$= Latest starting time of activity (i, j)

(Lf)= Latest finishing time of activity (i, j)

Determination of critical path

- Critical event – The event with zero slack times are called critical events. In other word the event j is said to be critical if E_j=L_j
- Critical activity – The activity with zero total float are known as critical activity.
- Critical path – The sequence of critical activities in a network is called critical path, and it defines the minimum time required to complete the project.

Problems:

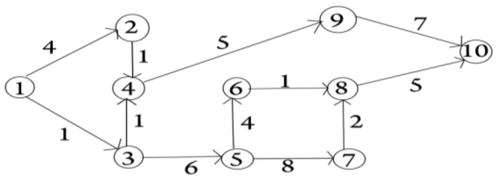
EXAMPLE 1:

A project has the following time schedule

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time(days)	4	1	1	1	6	5	4	8	1	2	5	7

Calculate the total float, critical activity, critical path and project duration.

Solution:



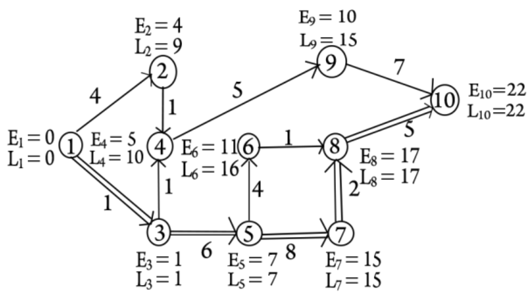
The network is T_E(Tail Event)=E_i, T_L(Head Event)= $L_i -$ $D_{ij}D_{ij}$(Duration)

Slack or Total float=T_L-T_E

Calculation table is as follows

Activity	D_{ij}	T_E	E_i+ D_{ij}	T_L	L_i	slack
1 – 2	4	0	4	5	9	5
1 – 3	1	0	1	0	1	0
2 – 4	1	4	5	9	10	5
3 – 4	1	1	2	9	10	8
3 – 5	6	1	7	1	7	0
4 – 9	5	5	10	10	15	5
5 – 6	4	7	11	12	16	5
5 – 7	8	7	15	7	15	0
6 – 8	1	11	12	16	17	5
7 – 8	2	15	17	15	17	0
8 − 10	5	17	22	17	22	0
9 − 10	7	10	17	15	22	5

The resultant network shows the critical path



From the table, the critical nodes are (1,3), (3,5), (5,7), (7,8) and (8,10).

From the table the critical path is $1 \to 3 \to 5 \to 7 \to 8 \to 10$

Project duration is 1+6+8+2+5=22

Therefore the project will be completed within 22 days.

EXAMPLE 2:

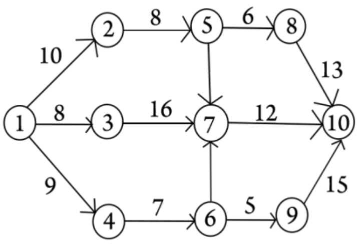
A project has the following time schedule

Activity	1-2	1-3	1-4	2-5	3-7	4-6	5-7	5-8	6-7	6-9	7-10	8-10	9-10
Time(days)	10	8	9	8	16	7	7	6	7	5	12	13	15

Find the critical path.

Solution:

The network is



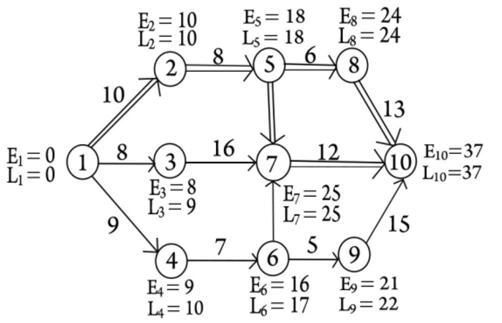
T_E(Tail Event)=E_i, T_L(Head Event)= $L_i - D_{ij} D_{ij}$(Duration)

Slack or Total float=T_L-T_E

Calculation table is as follows

Activity	D_{ij}	T_E	E_i+ D_{ij}	T_L	L_i	slack
1 – 2	10	0	10	0	10	0
1 – 3	8	0	8	1	9	1
1– 4	9	0	9	1	10	1
2– 5	8	10	18	10	18	0
3 – 7	16	8	24	9	25	1
4 – 6	7	9	16	10	17	1
6– 7	7	16	23	18	25	2
6– 9	5	16	21	17	22	1
7– 10	12	25	37	25	37	0
9– 10	15	21	36	22	37	1
5– 7	7	18	25	18	25	0
5– 8	6	18	24	18	24	0
8 – 10	13	24	37	24	37	0

The resultant network shows the critical path



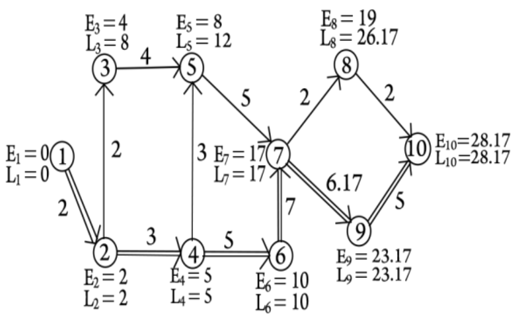
From the table, the critical nodes are (1,2), (2,5), (5,7), (7,10), (5,8) and (8,10).

From the table we get two possible critical paths, there are

i. $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10$

ii. $1 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10$

Further more examples are as under



Here critical path is

$1 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 7 \rightarrow 9 \rightarrow 10$

This implies eccentricity

IV CONCLUSION

A vertex v is called an eccentric vertex of u, if e (u) = d (u, v); where u, v ∈ G, i.e. v is a vertex that is farthest from u. If v is an eccentric vertex of some vertex of G such a vertex v is an eccentric vertex of the graph G . In other words, a vertex v is an eccentric vertex of G, if v is farthest from some vertex of G. The sequence of critical activities in a network is called critical path, and it defines the minimum time required to complete the project.

From this we conclude that eccentricity is equal to critical path.

REFERENCE

[1] Graph Theory and its Applications by Gross.JL and Yellen.J CRC Press LLC, 1998
[2] Graph Theory by Diestel.R Springer-Verlag 1997
[3] Introduction to Graph Theory by West.DB Prentice Hall 1996
[4] Introduction to Graph Theory by R.J. Wilson Addison Wesley Longman 1996
[5] Introduction to Graph Theory by R.J. Trudeau Dover Pubns 1994
[6] Graphs: An Introductory Approach by J. Wilson and J.J. Watkins John Wiley & Sons 1990
[7] Graph Theory by Gould.RJ Benjamin/Cummings 1988
[8] Introduction to Graph theory By Gary chartrand & Ping Zhang.